PB87-167003

Comparison of Numerical and Laboratory Experiments on Density-Stratified Flows Around a Three-Dimensional Hill

North Carolina State Univ. at Raleigh

Prepared for

Environmental Protection Agency
Research Triangle Park, NC

Mar 87

U.S. Department of Commerce
National Technical Information Service
NTIS

PB87-107003

EPA/600/D-87/055
March 1987

A COMPARISON OF NUMERICAL AND LABORATORY EXPERIMENTS ON DENSITY-STRATIFIED FLOWS AROUND A THREE-DIMENSIONAL HILL

by

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CR 811973

*On assignment from the National Oceanic and Atmospheric Administration, U.S. Department of Commerce

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OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
RESEARCH TRIANGLE PARK, NC 27711

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO. EPA/600/D-87/055	2.	3. RECIPIENT'S ACCESSION NO. PB87-167003/AS
4. TITLE AND SUBTITLE A Comparison of Numerical and Laboratory Experiments on Density-Stratified Flows around a Three-Dimensional Hill		5. REPORT DATE March 1987 6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) J.W. Rottman, R.E. Lawson, Jr. and W.H. Snyder		8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Atmospheric Sciences Research Laboratory Office of Research and Development U.S. Environmental Protection Agency Research Triangle Park, NC 27711		10. PROGRAM ELEMENT NO. A101/B/68/02-5033(FY-87) 11. CONTRACT/GRANT NO. Coop. Agmt. & INHOUSE CR-811973
12. SPONSORING AGENCY NAME AND ADDRESS Atmospheric Sciences Research Laboratory - RTP, NC Office of Research and Development U.S. Environmental Protection Agency Research Triangle Park, NC 27711		13. TYPE OF REPORT AND PERIOD COVERED 14. SPONSORING AGENCY CODE EPA/600/09
15. SUPPLEMENTARY NOTES		

16. ABSTRACT

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17. KEY WORDS AND DOCUMENT ANALYSIS		
a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group

18. DISTRIBUTION STATEMENT RELEASE TO PUBLIC	19. SECURITY CLASS *(This Report)* UNCLASSIFIED	21. NO. OF PAGES 13
	20. SECURITY CLASS *(This page)* UNCLASSIFIED	22. PRICE

EPA Form 2220-1 (Rev. 4-77) PREVIOUS EDITION IS OBSOLETE

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A COMPARISON OF NUMERICAL AND LABORATORY EXPERIMENTS ON DENSITY-STRATIFIED FLOWS AROUND A THREE-DIMENSIONAL HILL

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ABSTRACT

Direct comparisons between the results of a numerical model and laboratory experiments are presented for density-stratified flows around an isolated three-dimensional hill. The numerical model integrates the Navier-Stokes equations for incompressible stratified flow using a finite-difference scheme. Numerical calculations were made for Reynolds numbers (based on the hill height) of 100 and 400 and for Froude numbers between 0.5 and 2 as well as for neutrally stratified flow. The laboratory experiments were performed in a small (2.1m long) stratified towing tank, a large (25m long) stratified towing tank, and a meteorological wind tunnel. Experimental results are presented for Reynolds numbers between 100 and 40,000 and for the same Froude numbers as used in the numerical calculations. For the small towing tank the numerical model reproduced qualitatively the experimental results, although there were some substantial quantitative differences, particularly near the hill surface and at the larger value of the Reynolds number. The numerical model with a Reynolds number of 100 compared best with the large scale laboratory experiments, although there were substantial differences near the hill surface and in the wake of the hill.

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INTRODUCTION

The ultimate application of the work described in this paper is the prediction of surface concentration patterns due to plumes from sources located upstream of a three-dimensional hill in a density-stratified flow. To make such predictions requires knowledge of both the mean flow pattern around the hill and the statistical characteristics of the turbulence of the flow. Previous experimental work has shown that when topography is present the most important feature of the flow is the displacement by the topography of the mean streamline through the source. In the present paper, we are concerned with determining the mean flow pattern by solving numerically the Navier-Stokes equations in three-dimensions, taking into account the bottom topography.

The main purpose of the present work is to evaluate a particular numerical model by comparing its results with some detailed laboratory measurements of the mean flow of linearly stratified fluids around a particular three-dimensional hill. The numerical model is basically that developed by Mason and Sykes (1979, 1981). The hill is an idealized, axisymmetrical model of Cinder Cone Butte, an isolated hill near Boise, Idaho, which has been the subject of extensive field studies and laboratory simulations. The idealized hill has the mathematical representation $z=h(x,y)$ with

$$h(x,y) = H/[1+(x^2+ y^2)^2/L^4], \qquad (1)$$

where H is the hill height, L is the hill radius at half height, and (x,y,z) are the Cartesian coordinates of a point in space with z in the vertical direction.

Comparisons between the numerical calculations and laboratory experiments were made for three specific experimental arrangements. First a small towing tank was used in which both the Reynolds number $Re=UH/\nu$ and the Froude number $F=U/(NH)$ were matched exactly with the numerical model, where U is the free-stream speed, ν is the kinematic viscosity of the fluid and N is the (nearly constant) buoyancy frequency of the linearly stratified fluid. This provided an overall evaluation of the accuracy of the approximations made in the model. Computational constraints restricted the Reynolds number of the flow that could be computed to less than about 500. Second, a large towing tank was used in which mean plume trajectories were measured and compared with particle paths calculated using the flow field produced using the numerical model. And finally, some comparisons were made with hot-wire measurements of the mean streamwise component of velocity in a meteorological wind tunnel. These latter two laboratory experiments had Reynolds numbers on the order of 10^4, so in these cases the kinematic viscosity in the numerical model has to be thought of as a large eddy viscosity.

NUMERICAL PROCEDURE

The numerical scheme we used was essentially the same as that described by Mason and Sykes (1979, 1981), which in turn is based on the scheme originally developed by Williams (1969). The scheme solves a finite-difference approximation of the primitive form of the Navier-Stokes equations for incompressible stratified flow in the Boussinesq approximation.

The equations are solved on a stretched rectangular mesh. The method is of second order accuracy in both space and time, except at the lower boundary.

The novel feature of the Mason and Sykes scheme is the manner in which it treats the lower boundary condition over the topography. A stretched rectangular mesh was used that covered the entire computational domain, including the region under the hill. The viscous stress terms were modified in the region near the hill surface such that the stress at the hill surface was (to first order) equal to the stress that would exist with a no slip condition at the hill surface. Although this approach put mesh points in regions of the computational domain that were not in the physical flow domain, it permitted the use of a fast method for solving the Poisson equation for the pressure; this method works only on a rectangular mesh. The actual method used for solving the Poisson equation on a stretched rectangular mesh was that described by Farnell (1980). This produced a fairly fast method of solving the full set of equations.

For the remaining boundary conditions it was assumed that the flow is symmetrical about the streamwise centerplane (mainly for reasons of economy since only half the flow domain need be computed), that the upper and side boundaries were solid with free slip, and the upstream and downstream limits of the computational domain were 'flow-through'. The method used to compute the flow-through conditions was similar to that proposed by Orlanski (1976).

In the calculations described in this paper, the number of mesh points used was 38 in the streamwise direction, 19 in the cross-stream direction and 32 in the vertical direction. The size of the computational domain was only slightly larger than shown in the figures below. The calculations were initiated with a kinematically correct velocity field with an upstream Blausius profile and then integrated forward in time until a steady state was reached. Generally this required sufficient time for a particle moving at the free stream speed to pass through the entire streamwise length of the computational domain. The calculations were run on an IBM 3090-200 computer and typically required about two hours of CPU time.

EXPERIMENTAL PROCEDURE

Three distinct experimental arrangements were used to compare with the numerical calculations. The experimental procedures are described in the next three subsections. Because only the experiments in the small towing tank were done specifically for the present study, they will be described in more detail than those done in the other two experimental arrangements. The details of the large towing tank experiments and the wind tunnel experiments may be found in the cited publications.

Small Towing Tank

The small towing tank is made of acrylic plastic and has a working section 208cm long, 21cm wide and 10cm deep. It is a 1/12th scale replica of the large towing tank at the EPA Fluid Modeling Facility. The tank was filled to a depth of 9cm with linearly stratified salt water, with a buoyancy frequency $N = [(g/\rho)d\rho/dz]^{1/2}$ of either 1.2 or 4.7rad/s, where g is the acceleration due to gravity and ρ is the fluid density. The model hill, with H = 1.3cm and L = 6.7cm, was mounted on a flat baseplate and this whole assembly was turned upside down and towed along the surface of the water with the baseplate immersed a few millimeters. Tow speeds of 0.8cm/s and 3.0cm/s

were used. These parameters gave values of F = 0.5 and Re = 100 and 400. We also did experiments at the same values of Re but without stratification.

Two methods of flow visualization were used. A vertical wire positioned on the centerline at the upstream edge of the baseplate was used to generate vertical lines of hydrogen bubbles or blue dye at periodic intervals. The lines of blue dye were created by adding thymol blue to the fluid; a current through the wire caused a change of pH such that the color of the fluid in the vicinity of the wire changed. We found that the hydrogen bubble technique worked best for the faster flows and the thymol blue technique best for the slower flows. The current through the wire was pulsed at a periodic rate so that an image of the Lagrangian flow field over the entire centerplane was obtained. This image was photographed and the curves of bubbles or dye were digitized for later comparison with numerical simulations.

Numerical simulations of these images were obtained by introducing a vertical line of particles at the upstream edge of the computational domain and tracking these particles through the computer-generated mean flow field. An image that should have matched the experimental photographs was then produced by plotting the position of these particles at the same equally spaced time intervals at which the current in the wire was pulsed.

Large Towing Tank

The large towing tank is 25m long, 2.5m wide and 1.2m deep. As with the small tank, salt water was used for stratification, usually with N = 1.3rad/s. A hill of similar shape, with H = 15.5cm and L = 80.0cm, was used and towed in the same manner as in the small tank. The particular experiments we are interested in involved the measurement of mean trajectories of dye released from sources upstream of the hill. In these experiments, neutrally buoyant, dyed fluid was released nearly isokinetically at a constant rate upstream of the hill and stereographic photography was used to record the image of the these trajectories over the hill. These stereographic images were then digitized and analyzed to determine the three-dimensional coordinates of the trajectories. A more detailed description of these experiments is given by Snyder, Thompson and Shipman (1985) and Thompson and Shipman (1986).

These measured trajectories were simulated by calculating the paths of particles released at the source locations through the computer-generated mean flow field over the hill.

Meteorological Wind Tunnel

The meteorological wind tunnel has a test section that is 18m long, 3.7m wide and 2m high. A detailed description this facility is given by Snyder (1979). The particular measurements with which we compare our numerical calculations were made with hot wires and were of only the mean streamwise component of the velocity about the hill in a simulated neutrally stratified atmospheric boundary layer. The hill used in this study was identical to the one used in the large towing tank study described above. A complete description of the experimental techniques used is given by Lawson and Snyder (1987).

Contrary to the Lagrangian view of the flow field produced by the two previous experimental techniques, this experiment produces an Eulerian image of the velocity field that can be compared directly with a contour plot of the computed streamwise component of the velocity. However, in the computer simulations we did not attempt to reproduce the atmospheric boundary layer, so these comparisons can only offer qualitative comparisons with the numerical

model.

RESULTS

Although the scales are different for each experimental arrangement, the results have been scaled so that in the figures shown below the hill has the same dimensions as in the small towing tank. This was done because the numerical results, which required a large amount of computer time, were obtained specifically for the small towing tank experiments.

Small Towing Tank

The experimental results obtained in the small towing tank are compared with the numerical results in Fig. 1. Qualitatively the comparisons are fairly good; they are somewhat better for the Re = 100 flows than for the Re = 400 flows. The calculations show the same general behavior as in the experiments. In particular the jetting motion over the hill for the F = 0.5 flows as opposed the the fairly uniform velocity profile over the hill in the neutral flow is quite clearly shown. Quantitatively, however there are several discrepancies. The largest discrepancies are near the hill surface, where the numerical method is only first order accurate. Also, in the Re = 400 flows the computed particle line shapes vary substantially from the measured line shapes once they pass over the top of the hill. One reason for this may be that the vertical wire in the experiments was supported by a thin plastic bar that extended from the downstream edge of the base plate to the

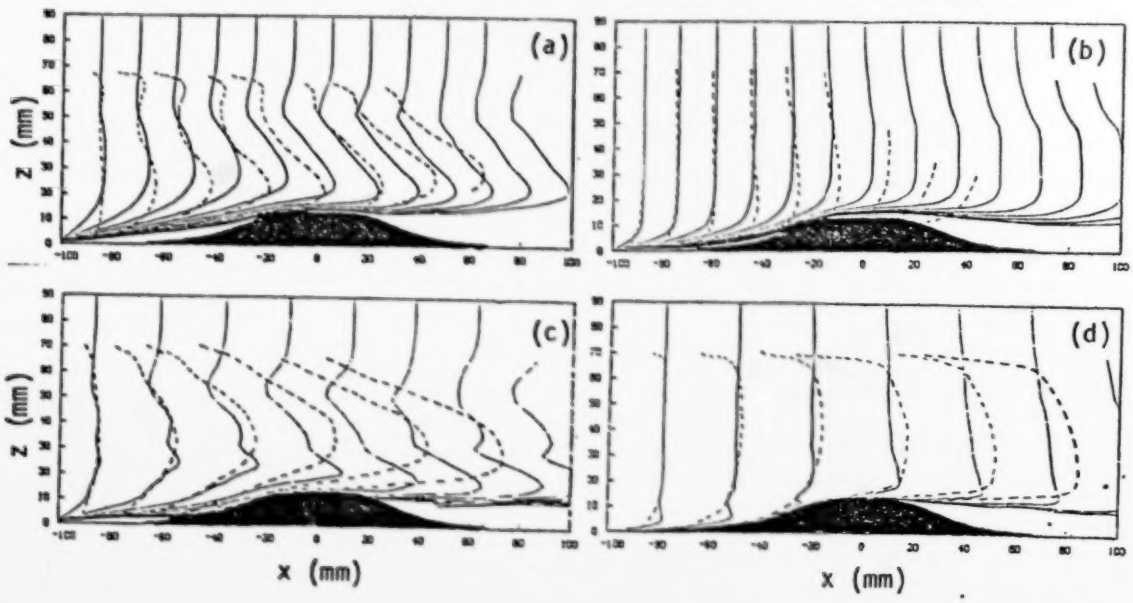

Fig. 1: Plots of an initially vertical line of particles released on the centerplane at the upstream edge of the baseplate at a number of equally spaced time intervals in a flow with Re = 100. The solid lines are the computed results and the dashed lines the experimentally measured results: (a) F = 0.5, Re = 100, (b) F = ∞, Re = 100, (c) F =0.5, Re = 400, (d) F = ∞, Re = 400.

upstream edge at about 70mm above the baseplate. This caused the measured particle lines to be retarded at the 70mm level and may have caused a speed up of the velocity in the region directly over the hill. It should be noted that comparisons of this type are very sensitive to any errors in the velocity field, because those errors are cumulative when computing the trajectories of particles through the velocity field.

Fig. 2 shows a projection onto a horizontal plane of the computed results for a horizontal line of particles released at half the hill height. No comparable experiments were done for this particular arrangement. According to Snyder et al. (1985), a wide variety of experiments verify that the so-called dividing streamline height is given by the expression d=(1-F)H, when F < 1. Streamlines originating below d go around the hill while those originating above d go over. The calculations seem to reproduce these results; in the neutral flow the lines of particles flow over the top of the hill, whereas in the flow with F=0.5 the particles flow around the hill.

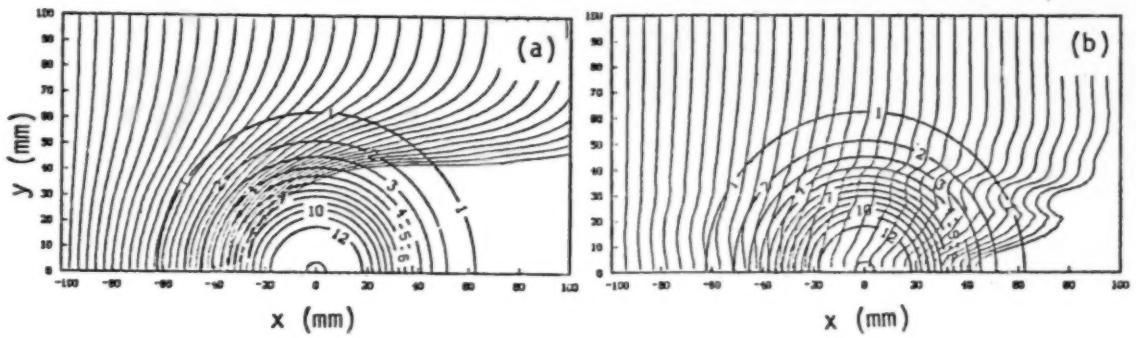

Fig. 2: Plots of the computed results for an initially horizontal line of particles released at the upstream edge of the hill at half the hill height at a number of equally spaced time intervals in a flow with Re = 100. The numbered semicircular lines are the hill contours in mm (the flow is assumed symmetrical about y=0 so only half the flow field is shown): (a) F = 0.5, (b) F = ∞.

Large Towing Tank

The comparisons of the numerical and experimental results for mean particle trajectories are shown in Figs. 3 and 4. Fig. 3 shows three trajectories from sources located upstream of the hill on the centerplane at different heights for Froude numbers 1, 2, and ∞. Even though the Reynolds numbers of the experimental flows were much larger than those for the computed flows, the measured mean trajectories were reasonably well predicted by the numerical model, although there are some discrepancies.The calculations did not show as strong a dip in the trajectories on the lee side of the hill as was observed in the experiments.

Fig. 4 shows the projections onto a horizontal plane of the numerical and experimental trajectories from sources located at one quarter and one half of the hill height and at several positions offset from the centerplane. It is apparent from the experimental observations that the lateral displacement of the trajectories increases as the Froude number decreases. The computational results show the same tendencies, but not as strongly as indicated by the experimental results. The comparisons get worse the closer

the sources are to the surface. There are particularly bad discrepancies on the lee side of the hill for the smaller Froude numbers, indicating that the wake behind the hill observed in the experiments was not reproduced by the calculations.

We also made comparisons with experiments in which the sources were located at the hill height, although we have not shown these comparisons here. The numerical results for the Re = 100 computer-generated flow field compared much better than the results for sources located below the hill height. It appeared that the numerical model did better the farther the trajectories were from the hill surface.

The comparisons with the measured trajectories when the Re = 400 computer-generated flow fields were used were much worse than the comparisons shown in Figs. 3 and 4.

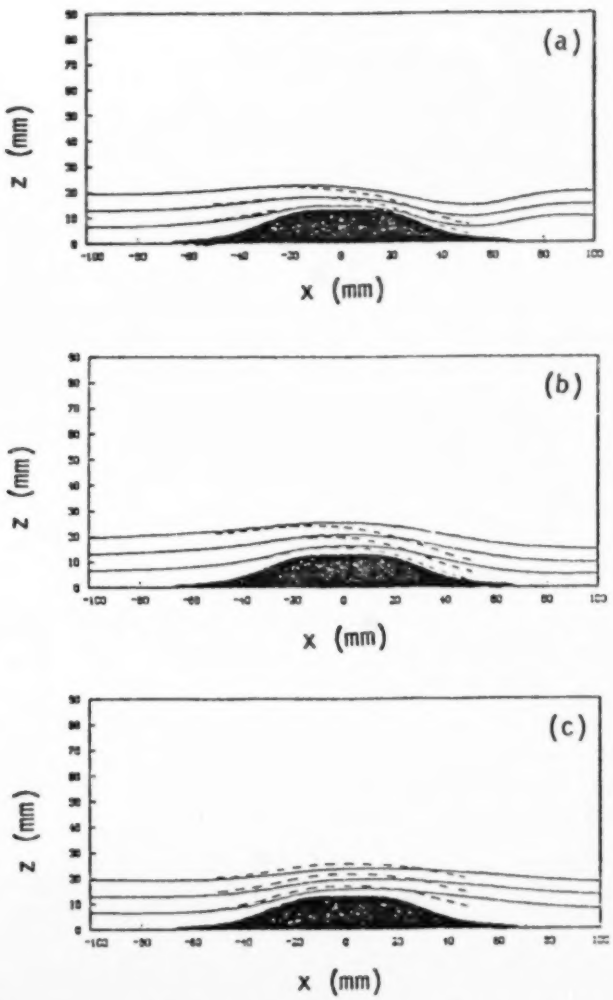

Fig. 3: Plots of mean particle trajectories from sources located on the centerplane and at heights 0.5, 1.0, and 1.5 times the hill height. The solid lines are the computed results at Re = 100 and the dashed lines the experimentally measured results: (a) F = 1.0, (b) F = 2.0, (c) F = ∞.

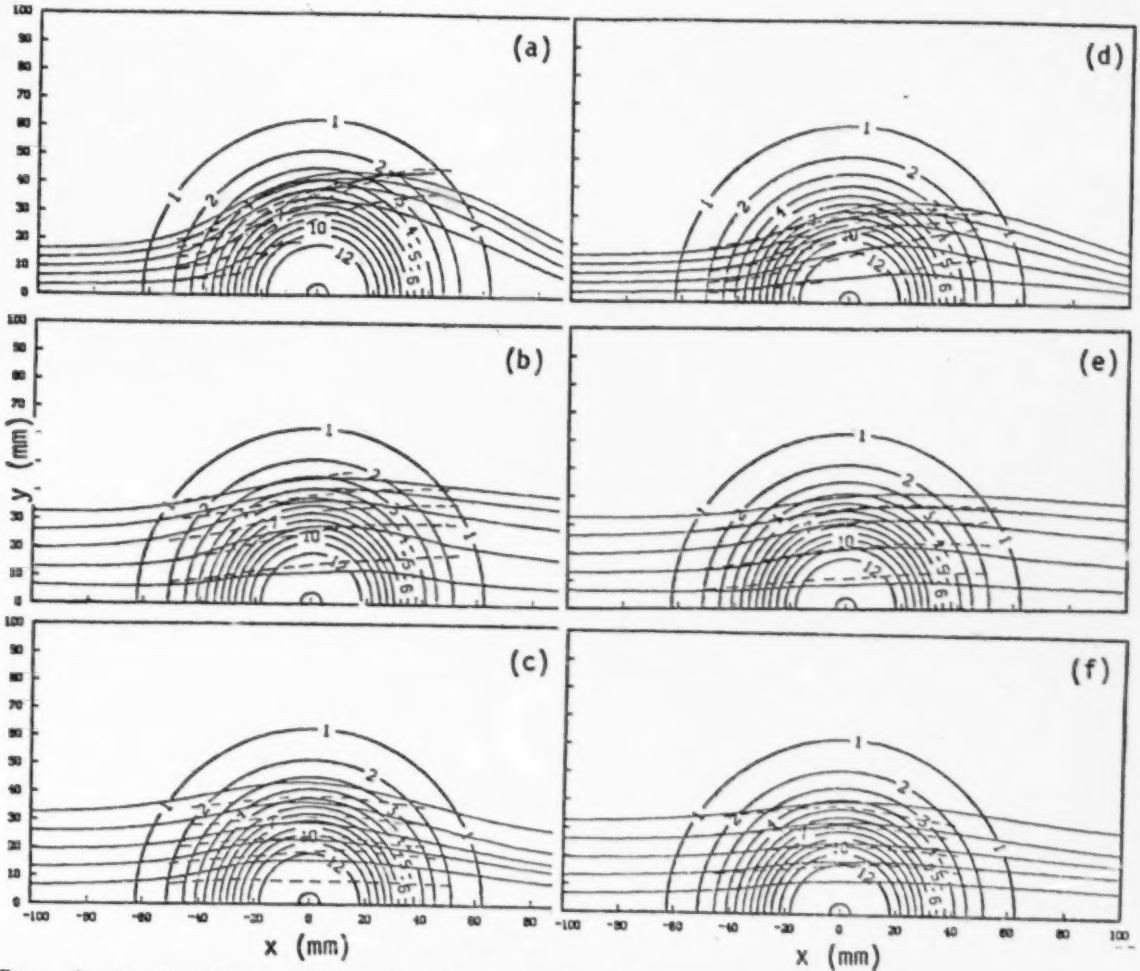

Fig. 4: Projections onto a horizontal plane of mean particle trajectories from sources located at one quarter of the hill height, (a)-(c), and at one half of the hill height, (d)-(f). The solid lines are the computed results at Re = 100 and the dashed lines the experimentally measured results: (a), (d) F = 1.0; (b), (e) F = 2.0; (c), (f) F = ∞.

Meteorological Wind Tunnel

Fig. 5 shows the comparison of the numerical results with the hot-wire measurements of the streamwise component of the velocity in the meteorological wind tunnel. The figure contains contour plots of the experimental results (at Re ≅ 40,000) and the numerical calculations at Re = 100. Although the results are not identical, there is a strong qualitative agreement between the two plots, including the region of speed up over the top of the hill and the recirculation region in the near lee side of the hill. It should be kept in mind that the wind tunnel experiments simulated a neutral atmospheric boundary layer, which was much thicker than the height of the hill, and no attempt was made to simulate this boundary layer in the numerical calculations. The computer-generated flow field with Re = 400 compares much worse with the experimentally measured flow field than does the Re = 100 flow field shown here.

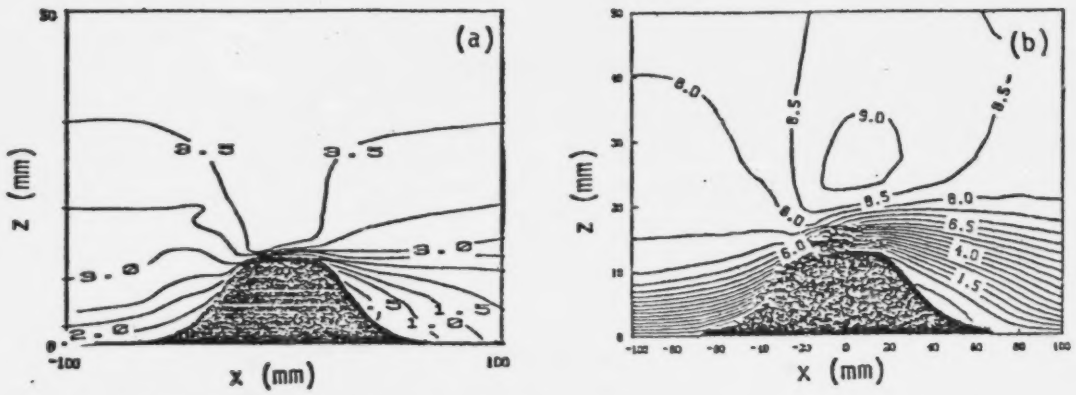

Fig. 5: Contour plots of the streamwise component of the velocity on the centerplane: (a) experimental results at Re = 40,000, (the contours are numbered in m/s with U=4m/s); (b) computed results at Re = 100, (the contours are numbered in mm/s with U = 7.7mm/s). Note that for clarity the vertical scale has been exaggerated by a factor of three.

CONCLUSIONS

We have carried out a limited comparison of the results of a three-dimensional numerical model for calculating stratified flows over an isolated hill with some detailed experimental measurements. The comparisons are limited mainly because of the large amount and expense of computer time necessary to make the calculations, but also because detailed experimental measurements of three-dimensional flows in the laboratory are also difficult and time consuming. A more extensive comparison must await for both an increase in computer processing speed and better methods for measuring velocity fields in the laboratory.

The results we have presented indicate that the numerical model qualitatively reproduces the experimental flow fields when the Reynolds number and Froude numbers are matched exactly but that there are some substantial quantitative differences. These differences are most prominent near the hill surface and at the larger value of the Reynolds number for which the calculations were performed. It may be that better agreement would be obtained if the lower boundary condition was handled differently. Of course, with greater computer storage and speed we could use a finer mesh that would supposedly give better agreement. On the other hand, all that may be needed is a different amount of stretching of the grid and a better representation of the upstream velocity profile at the upstream boundary of the computational domain. The expense of the computations prevented us from exploring all these different possibilities.

The comparisons with the large scale laboratory experiments showed quite clearly that the computed flow with Re = 100 did a much better job of reproducing the measured flow fields than did the calculations with Re = 400. It may be that a computed flow field with Re = 50 may even do a better job, but we have not explored this possibility. The greatest quantitative discrepancies in the comparisons were the regions of the flow close to the hill surface and particularly in the wake of the hill.

ACKNOWLEDGEMENTS

We thank R.I. Sykes for his help with adapting the Mason & Sykes computer program to our problem. We gratefully acknowledge the help of the entire staff at the U.S. Environmental Protection Agency's Fluid Modeling Facility and in particular G.L. Marsh, M.S. Shipman and R.S. Thompson. The first author was supported by the U.S. Environmental Protection Agency through the Cooperative Agreement CR-811973 with North Carolina State University.

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